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SUPER PERFECT NUMBERS AND

SUPER TWIN PRIMES

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1. Perfect numbers with translation parameter

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By $\sigma(a)$ we denote the number of factors of a natural number a.

If a natural number a satisfies $\sigma(a) = 2a$, then it is called a perfect number due to Greek mathematician Euclid.

$a = 6, 28, 496, 8128,$ are perfect numbers .

The final result in Element by Euclid states that if $q = 2^{e+1} - 1$ is a prime,then $a = 2^e q$ turns out to be a perfect number.

Given an integer m , perfect numbers α with translation parameter m are defined to be a natural number α satisfying $\sigma(\alpha) = 2\alpha - m$.

If $m = 0$ and α is even , it is written as $\alpha = 2^e q$, where $q = 2^{e+1} - 1$ is a prime. This was proved by L.Euler.

2. SUPER PERFECT NUMBERS WITH TRANSLATION PARAMETER m

It is shown that if $q = 2^{e+1} - 1 + m$ is prime , then $a = 2^e$ satisfies $\sigma(\sigma(a) + m) = 2a + m$.

In general, natural numbers a are said to be super perfect numbers with translation parameter m, if they satisfy $\sigma(\sigma(a) + m) = 2a + m$.

If $m = 0$ then the equation turns out to be $\sigma^2(a) = 2a$, which looks like the equation $\sigma(a) = 2a$ defining perfect numbers. In this case, a is said to be super perfect number. This notion was introduced by D.Suryanaryana in 1969.

He proves that even super perfect numbers turn out to be 2^e where $q = 2^{e+1} - 1$ is a prime. So $2^e q$ is perfect.

Table 1. solutions of $\sigma^2(a) = 2a, q = 2a - 1$ (super perfect numbers)

a	factor of a	q	factor of q
2	2	3	3
4	2^2	7	7
16	2^4	31	31
64	2^6	127	127
4096	2^{12}	8191	8191
65536	2^{16}	131071	131071

Here, q is a Mersenne prime.

TABLE 2. solutions when $m = -18$

a	factors of	$p = a/3$	$q = 2p - 7$
15	$3 * 5$	5	3
16	2^4		
21	$3 * 7$	7	7
27	3^3	9	
39	$3 * 13$	13	19
57	$3 * 19$	19	31
64	2^6		
111	$3 * 37$	37	67
129	$3 * 43$	43	79
201	$3 * 67$	67	127
219	$3 * 73$	73	139
237	$3 * 79$	79	151
309	$3 * 103$	103	199
327	$3 * 109$	109	211
417	$3 * 139$	139	271
471	$3 * 157$	157	307
579	$3 * 193$	193	379

2.1. **super perfect numbers with translation parameter $m = -18$.**

proposition 1. *When* $m = -18$, *if* $a = 3p($ *where* p *is prime* $, \neq 3)$ *and* $q = 2p - 7$ *is also prime, then* $a = 3p$ *turns out to be super perfect numbers with translation parameter* $m = -18$.

Conversely,

proposition 2. *if* $a = 3L(L \neq 0 \bmod 7)$ *is super perfect numbers with translation parameter* $m = -18$*,then both* L *and* $q = 2L - 7$ *are prime.*

Actually, since both p and $q = 2p - 7$ are prime, they are called super twin primes.

3. SUPER TWIN PRIME CONJECTURE

Given integers $(a > 0, b)$, if both q and $p = aq + b$ are primes, then the pair $(p, q)$is said to be super twin primes with respect to (a, b).

If $a = 1, b = 2$ then super twin primes turns out to be just twin primes.

No one knows that there exist infinitly many twin primes .

Suppose that if (i) $a + b \equiv 1 \mod 2$ and (ii) a, b are relatively prime.

It seems that there exist infinitly many super twin primes $(q, p = aq + b)$. ???

This conjecture was proposed by Hiroto Takahashi in March 2018. Then he was 10 years old, attending elementary school, every day.

4. Ultra perfect number of type II

When $a = 2^e, (q = 2a-1+m : \text{prime})$,putting $N = 2^{e+1}-1$, from $\sigma(a) = N = 2a-1 = q-m$, it follws that $q = \sigma(a)+m$.

Using $A = \sigma(a) + m$, $\sigma(A) = q + 1 = A + 1$. Putting $B = \sigma(A) - 1$, we get $\sigma(B) = q + 1 = 2a + m$.

Under the assumption $a = 2^e, (q = 2a - 1 + m : \text{prime})$, suppose that a satisfies the simultaneous equations

$A = \sigma(a) + m$, $B = \sigma(A) - 1$, $\sigma(B) = 2a + m$.

Then a are said to be ultra perfect number of type II with with translation parameter m. (Forgetting $a = 2^e$.)

When $m = -28, -18, -14, -58$, ultra perfect numbers of type II generate ultra triplet primes.

Table 3. $m = -18$ ultra perfect number of type II

a	$factor$	$p = a/3$	$q = 2p - 7$	$r = 6p - 19$	$r + 1 = 2a + m$
15	$3 * 5$	5	3	11	12
16	2^4	5.333333333	3.666666667	13	14
21	$3 * 7$	7	7	23	24
29	29	9.666666667	12.33333333	39	40
39	$3 * 13$	13	19	59	60
64	2^6	21.33333333	35.66666667	109	110
129	$3 * 43$	43	79	239	240
201	$3 * 67$	67	127	383	384
219	$3 * 73$	73	139	419	420
309	$3 * 103$	103	199	599	600
669	$3 * 223$	223	439	1319	1320
729	3^6	243	479	1439	1440

5. Ultra triplet primes

theorem 1. *When $m = -18$, if $p, q = 2p-7, r = 6p-19$ are all primes, then $a = 3p$ turn out to be ultra perfect numbers of type II .*

Such (p, q, r) are said to be ultra triplet primes.

Given integers $(a > 0, b, c > 0, d)$, if $q, p = aq + b, r = cq + d$ are all primes then (p, q, r) を are said to be ultra triplet primes with respect to a, b, c, d.

Suppose that if (i) $a + b, c + d \equiv 1 \mod 2$ and (ii) a, b and c, d are both relatively prime.

Moreover, $ac \equiv -bd \not\equiv 0 \mod 3$ (by H.Mizutani).

It seems that there exist infinitly many ultra triplet primes $(q, p = aq + b, r = cq + d)$ (By H.Takahashi).