この研究では、インド式計算法の一般化について考える。

$99^2 = 9801$

$999^2 = 998001$

$9999^2 = 99980001$

9がn個並んだ数を $9^{(n)}$ と書くことにする。

$(9^{(n)})^2 = 9^{(n-1)}8^{(1)}0^{(n-1)}1^{(1)}$

$(9^{(n)})^2 = 9^{(n-1)}8^{(1)}0^{(n-1)}1^{(1)}$であることの証明

証明）

$a = 1 = 0.9^{(\infty)}$

$0.9^{(n)} = a - \frac{a}{10^n}$

$0.9^{(n)} = \alpha$とする。

$$\begin{aligned}\alpha^2 &= (a^2 - \frac{a^2}{10^n}) - (\frac{a^2}{10^n} - \frac{a^2}{10^{2n}}) \\ &= (0.9^{\infty} - 0.0^{(n)}9^{(\infty)}) - (0.0^{(n)}9^{(\infty)} - 0.0^{(2n)}9^{(\infty)}) \\ &= 0.9^{(n)} - 0.0^{(n)}9^{(n)} \\ &= 0.9^{(n-1)}8^{(1)}0^{(n-1)}1^{(1)}\end{aligned}$$

$$(1^{(10)})^2 = [1, 2, 3, 4, 5, 6, 7, 9, 0, 0, 9, 8, 7, 6, 5, 4, 3, 2, 1]$$
$$(2^{(10)})^2 = [4, 9, 3, 8, 2, 7, 1, 6, 0, 3, 9, 5, 0, 6, 1, 7, 2, 8, 4]$$
$$(3^{(10)})^2 = [1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 8, 8, 8, 8, 8, 8, 8, 8, 8, 9]$$
$$(4^{(10)})^2 = [1, 9, 7, 5, 3, 0, 8, 6, 4, 1, 5, 8, 0, 2, 4, 6, 9, 1, 3, 6]$$
$$(5^{(10)})^2 = [3, 0, 8, 6, 4, 1, 9, 7, 5, 2, 4, 6, 9, 1, 3, 5, 8, 0, 2, 5]$$
$$(6^{(10)})^2 = [4, 4, 4, 4, 4, 4, 4, 4, 4, 3, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6]$$
$$(7^{(10)})^2 = [6, 0, 4, 9, 3, 8, 2, 7, 1, 4, 8, 3, 9, 5, 0, 6, 1, 7, 2, 9]$$
$$(8^{(10)})^2 = [7, 9, 0, 1, 2, 3, 4, 5, 6, 6, 3, 2, 0, 9, 8, 7, 6, 5, 4, 4]$$
$$(9^{(10)})^2 = [9, 9, 9, 9, 9, 9, 9, 9, 9, 8, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]$$

$(3^{(10)})^3 =$

[3, 7, 0, 3, 7, 0, 3, 7, 0, 2, 5, 9, 2, 5, 9, 2, 5, 9, 2, 7, 0, 3, 7, 0, 3, 7, 0, 3, 7]

$(6^{(10)})^3 =$

[2, 9, 6, 2, 9, 6, 2, 9, 6, 2, 0, 7, 4, 0, 7, 4, 0, 7, 4, 1, 6, 2, 9, 6, 2, 9, 6, 2, 9, 6]

$(9^{(10)})^3 =$

[9, 9, 9, 9, 9, 9, 9, 9, 9, 7, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9]

$$(9^{(10)})^4 =$$

$$[9, 9, 9, 9, 9, 9, 9, 9, 9, 6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 5, 9, 9, 9, 9, 9, 9, 9, 9, 9, 6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]$$

$$(9^{(10)})^5 =$$

$$[9, 9, 9, 9, 9, 9, 9, 9, 9, 5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9]$$

$$(9^{(10)})^6 =$$

$$[9, 9, 9, 9, 9, 9, 9, 9, 9, 4, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 9, 9, 9, 9, 9, 9, 9, 9, 8, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 9, 9, 9, 9, 9, 9, 9, 9, 9, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]$$

$(9^{(10)})^7 =$

$$[9, 9, 9, 9, 9, 9, 9, 9, 9, 3, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 9, 9, 9, 9, 9, 9, 9, 9, 6, 5, 0, 0, 0, 0, 0, 0, 0, 0, 3, 4, 9, 9, 9, 9, 9, 9, 9, 9, 7, 9, 0, 0, 0, 0, 0, 0, 0, 0, 0, 6, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9]$$

$(9^{(10)})^8 =$

$$[9, 9, 9, 9, 9, 9, 9, 9, 9, 2, 0, 0, 0, 0, 0, 0, 0, 0, 2, 7, 9, 9, 9, 9, 9, 9, 9, 9, 4, 4, 0, 0, 0, 0, 0, 0, 0, 0, 6, 9, 9, 9, 9, 9, 9, 9, 9, 9, 4, 4, 0, 0, 0, 0, 0, 0, 0, 0, 2, 7, 9, 9, 9, 9, 9, 9, 9, 9, 9, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]$$

$(9^{(10)})^9 =$

$$[9, 9, 9, 9, 9, 9, 9, 9, 9, 1, 0, 0, 0, 0, 0, 0, 0, 0, 3, 5, 9, 9, 9, 9, 9, 9, 9, 9, 1, 6, 0, 0, 0, 0, 0, 0, 0, 1, 2, 5, 9, 9, 9, 9, 9, 9, 9, 8, 7, 4, 0, 0, 0, 0, 0, 0, 0, 0, 8, 3, 9, 9, 9, 9, 9, 9, 9, 9, 6, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 8, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9]$$

$(1^{(10)})^9 =$

$$[2, 5, 8, 1, 1, 7, 4, 7, 8, 9, 3, 9, 0, 1, 3, 9, 8, 7, 0, 3, 7, 7, 3, 7, 6, 9, 6, 9, 1, 2, 5, 6, 5, 7, 1, 6, 1, 6, 4, 9, 4, 9, 6, 5, 4, 9, 9, 8, 4, 3, 2, 4, 6, 5, 4, 5, 4, 8, 8, 8, 2, 1, 5, 7, 8, 5, 6, 1, 2, 8, 2, 5, 4, 4, 8, 6, 8, 1, 9, 5, 9, 1]$$

$(9^{(10)})^9 =$

$$[9, 9, 9, 9, 9, 9, 9, 9, 9, 1, 0, 0, 0, 0, 0, 0, 0, 0, 3, 5, 9, 9, 9, 9, 9, 9, 9, 9, 1, 6, 0, 0, 0, 0, 0, 0, 0, 1, 2, 5, 9, 9, 9, 9, 9, 9, 9, 8, 7, 4, 0, 0, 0, 0, 0, 0, 0, 0, 8, 3, 9, 9, 9, 9, 9, 9, 9, 9, 6, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 8, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9]$$

$(9^{(n)})^m$は格別にきれいに数が並ぶ。
その理由として$0.9^{(\infty)} = 1$であることが関係していると考えられる。
だから一般に、Gを底とし、G-1＝gとするとき
$0.g^{(\infty)} = 1$であることを使って計算するので
$(g^{(n)})^m$も割と簡単に書けることが予想できます。

$$2 \leq G \leq 10, n = 10, (g^{(n)})^2$$

$(1^{(10)})^2 = [1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]$

$(2^{(10)})^2 = [2, 2, 2, 2, 2, 2, 2, 2, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]$

$(3^{(10)})^2 = [3, 3, 3, 3, 3, 3, 3, 3, 3, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]$

$(4^{(10)})^2 = [4, 4, 4, 4, 4, 4, 4, 4, 4, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]$

$(5^{(10)})^2 = [5, 5, 5, 5, 5, 5, 5, 5, 5, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]$

$(6^{(10)})^2 = [6, 6, 6, 6, 6, 6, 6, 6, 6, 5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]$

$(7^{(10)})^2 = [7, 7, 7, 7, 7, 7, 7, 7, 7, 6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]$

$(8^{(10)})^2 = [8, 8, 8, 8, 8, 8, 8, 8, 8, 7, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]$

$(9^{(10)})^2 = [9, 9, 9, 9, 9, 9, 9, 9, 9, 8, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]$

$(9^{(10)})^2$ =[9, 9, 9, 9, 9, 9, 9, 9, 9, 8, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]

２乗計算公式

$G \geq 2, G - 1 = g$ のとき、

$$(g^{(n)})^2 = g^{(n-1)}(g-1)^{(1)}0^{(n-1)}1^{(1)}$$ が成り立つ。

$$2 \leq G \leq 10, n = 10, (g^{(n)})^3$$

$$(1^{(10)})^3 = [1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$$

$$(2^{(10)})^3 = [2, 2, 2, 2, 2, 2, 2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2]$$

$$(3^{(10)})^3 = [3, 3, 3, 3, 3, 3, 3, 3, 3, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3]$$

$$(4^{(10)})^3 = [4, 4, 4, 4, 4, 4, 4, 4, 4, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4]$$

$$(5^{(10)})^3 = [5, 5, 5, 5, 5, 5, 5, 5, 5, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]$$

$$(6^{(10)})^3 = [6, 6, 6, 6, 6, 6, 6, 6, 6, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6]$$

$$(7^{(10)})^3 = [7, 7, 7, 7, 7, 7, 7, 7, 7, 5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7]$$

$$(8^{(10)})^3 = [8, 8, 8, 8, 8, 8, 8, 8, 8, 6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8]$$

$$(9^{(10)})^3 = [9, 9, 9, 9, 9, 9, 9, 9, 9, 7, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9]$$

$(9^{(10)})^3 = [9, 9, 9, 9, 9, 9, 9, 9, 9, 7, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9]$

３乗計算公式

$G \geq 3$、$G - 1 = g$のとき、

$(g^{(n)})^3 = g^{(n-1)}(g-2)^{(1)}0^{(n-1)}2^{(1)}g^{(n)}$が成り立つ

$$2 \leq G \leq 10, n = 10, (g^{(n)})^4$$

$(1^{(10)})^4$ = [1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]

$(2^{(10)})^4$ = [2, 2, 2, 2, 2, 2, 2, 2, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]

$(3^{(10)})^4$ = [3, 3, 3, 3, 3, 3, 3, 3, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 3, 3, 3, 3, 3, 3, 3, 3, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]

$(4^{(10)})^4$ = [4, 4, 4, 4, 4, 4, 4, 4, 4, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 4, 4, 4, 4, 4, 4, 4, 4, 4, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]

$(5^{(10)})^4$ = [5, 5, 5, 5, 5, 5, 5, 5, 5, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]

$(6^{(10)})^4$ = [6, 6, 6, 6, 6, 6, 6, 6, 6, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 5, 6, 6, 6, 6, 6, 6, 6, 6, 6, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]

$(7^{(10)})^4$ = [7, 7, 7, 7, 7, 7, 7, 7, 7, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 5, 7, 7, 7, 7, 7, 7, 7, 7, 7, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]

$(8^{(10)})^4$ = [8, 8, 8, 8, 8, 8, 8, 8, 8, 5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 5, 8, 8, 8, 8, 8, 8, 8, 8, 8, 5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]

$(9^{(10)})^4$ = [9, 9, 9, 9, 9, 9, 9, 9, 9, 6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 5, 9, 9, 9, 9, 9, 9, 9, 9, 9, 6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]

$(9^{(10)})^4$ =

[9, 9, 9, 9, 9, 9, 9, 9, 9, 6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 5, 9, 9, 9, 9, 9, 9, 9, 9, 9, 6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]

４乗計算公式

$G \geq 6$、$G - 1 = g$のとき、

$$(g^{(n)})^4 = g^{(n-1)}(g-3)^{(1)}0^{(n-1)}5^{(1)}g^{(n-1)}(g-3)^{(1)}0^{(n-1)}1^{(1)}$$

が成り立つ。

$$2 \leq G \leq 10, n = 10, (g^{(n)})^5$$

$(1^{(10)})^5$ = [1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

$(2^{(10)})^5$ = [2, 2, 2, 2, 2, 2, 2, 2, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 2, 2, 2, 2, 2, 2, 1, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2]

$(3^{(10)})^5$ = [3, 3, 3, 3, 3, 3, 3, 3, 2, 3, 0, 0, 0, 0, 0, 0, 0, 0, 2, 1, 3, 3, 3, 3, 3, 3, 3, 3, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3]

$(4^{(10)})^5$ = [4, 4, 4, 4, 4, 4, 4, 4, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 4, 4, 4, 4, 4, 4, 4, 4, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4]

$(5^{(10)})^5$ = [5, 5, 5, 5, 5, 5, 5, 5, 5, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 5, 5, 5, 5, 5, 5, 5, 5, 4, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]

$(6^{(10)})^5$ = [6, 6, 6, 6, 6, 6, 6, 6, 6, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 6, 6, 6, 6, 6, 6, 6, 6, 5, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6]

$(7^{(10)})^5$ = [7, 7, 7, 7, 7, 7, 7, 7, 7, 3, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 7, 7, 7, 7, 7, 7, 7, 7, 6, 6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7]

$(8^{(10)})^5$ = [8, 8, 8, 8, 8, 8, 8, 8, 8, 4, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 8, 8, 8, 8, 8, 8, 8, 8, 7, 8, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8]

$(9^{(10)})^5$ = [9, 9, 9, 9, 9, 9, 9, 9, 9, 5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9]

$(9^{(10)})^5$ =

[9, 9, 9, 9, 9, 9, 9, 9, 9, 5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9]

５乗計算公式

$G \geq 10$、$G - 1 = g$ のとき、

$(g^{(n)})^5$ =

$$g^{(n-1)}(g-4)^{(1)}0^{(n-1)}9^{(1)}g^{(n-1)}(g-9)^{(1)}0^{(n-1)}4^{(1)}g^{(n)}$$

が成り立つ。

２乗計算公式

$G-1=g$、$a=0.g^{(\infty)}$とする。

$0.g^{(n)}=\alpha$とおくと$\alpha=a-\frac{a}{G^n}$と書ける。

$\alpha^2=(a^2-\frac{a^2}{G^n})-(\frac{a^2}{G^n}-\frac{a^2}{G^{2n}})$

$a^4=1=0.9^{(\infty)}$

$\alpha^2=0.g^{(n)}(g-1)^{(1)}0^{(n-1)}1^{(1)}$

よって$G\geq 2$のとき$(g^{(n)})^2=g^{(n)}(g-1)^{(1)}0^{(n-1)}1^{(1)}$が成り立つ。

３乗計算公式

$G-1=g$、$a=0.g^{(\infty)}$とする。

$0.g^{(n)}=\alpha$とおくと、$\alpha=a-\frac{a}{G^n}$と書ける。

$\alpha^3=(a^3-\frac{a^3}{G^n})-2(\frac{a^3}{G^n}-\frac{a^3}{G^{2n}})+(\frac{a^3}{G^{2n}}-\frac{a^3}{G^{3n}})$

$a^3=1=0.g^{(\infty)}$

$\alpha^3=0.g^{(n-1)}(g-2)^{(1)}0^{(n-1)}2^{(1)}g^{(n)}$

よって$G\geq 3$のとき$(g^{(n)})^3=g^{(n-1)}(g-2)^{(1)}0^{(n-1)}2^{(1)}g^{(n)}$が成り立つ。

４乗計算公式

$G-1=g$、$a=0.g^{(\infty)}$とする。

$0.g^{(n)}=\alpha$とおくと、$\alpha=a-\frac{a}{G^n}$と書ける。

$\alpha^4=(a^4-\frac{a^4}{G^n})-3(\frac{a^4}{G^n}-\frac{a^4}{G^{2n}})+3(\frac{a^4}{G^{2n}}-\frac{a^4}{G^{3n}})-(\frac{a^4}{G^{3n}}-\frac{a^4}{G^{4n}})$

$a^4=1=0.g^{(\infty)}$

$\alpha^4=0.g^{(n-1)}(g-3)^{(1)}0^{(n-1)}5^{(1)}g^{(n-1)}(g-3)^{(1)}0^{(n-1)}1^{(1)}$

よって$G\geq 6$のとき$(g^{(n)})^4=g^{(n-1)}(g-3)^{(1)}0^{(n-1)}5^{(1)}g^{(n-1)}(g-3)^{(1)}0^{(n-1)}1^{(1)}$

が成り立つ。

５乗計算公式

$G-1=g$、$a=0.g^{(\infty)}$とする。

$0.g^{(n)}=\alpha$とおくと、$\alpha=a-\frac{a}{G^n}$と書ける。

$\alpha^5=(a^5-\frac{a^5}{G^n})-4(\frac{a^5}{G^n}-\frac{a^5}{G^{2n}})+6(\frac{a^5}{G^{2n}}-\frac{a^5}{G^{3n}})-4(\frac{a^5}{G^{3n}}-\frac{a^5}{G^{4n}})+(\frac{a^5}{G^{4n}}-\frac{a^5}{G^{5n}})$

$a^5=1=0.g^{(\infty)}$

$\alpha^5=0.g^{(n-1)}(g-4)^{(1)}0^{(n-1)}9^{(1)}g^{(n-1)}(g-9)^{(1)}0^{(n-1)}4^{(1)}g^{(n)}$

よって$G\geq 10$のとき$(g^{(n)})^5=g^{(n-1)}(g-4)^{(1)}0^{(n-1)}9^{(1)}g^{(n-1)}(g-9)^{(1)}0^{(n-1)}4^{(1)}g^{(n)}$

が成り立つ。

分割和

$G \geq 2$、$G-1 = g$ のとき、

$(g^{(n)})^2 = g^{(n-1)}(g-1)^{(1)}0^{(n-1)}1^{(1)}$ の右辺の 2 分割和について考えました。

右辺を $g^{(n-1)}(g-1)^{(1)}$ と $0^{(n-1)}1^{(1)}$ に分割して足すと

$g^{(n-1)}(g-1)^{(1)} + 0^{(n-1)}1^{(1)} = g^{(n)}$ となります。

例）

$G = 10$、$n = 5$ の場合

$(9^{(5)})^2 = 9^{(4)}8^{(1)}0^{(4)}1^{(1)}$

$99998 + 00001 = 99999 = 9^{(5)}$

3乗計算公式の3分割和についての例

G = 10、n = 5の場合

$(9^{(5)})^3 = 9^{(4)}7^{(1)}0^{(4)}2^{(1)}9^{(5)}$

$99997 + 00002 + 99999 = 199998 = 2 \times 99999 = 2 \times 9^{(5)}$

2乗計算公式の2分割和 = $g^{(n)}$

3乗計算公式の3分割和 = $2 \times g^{(n)}$

4乗計算公式の4分割和 = $2 \times g^{(n)}$

5乗計算公式の5分割和 = $3 \times g^{(n)}$