第2種スターリング数の法nについての周期性

村田 洋輔
学習院大学理学部数学科4年

平成22年2月1日

目 次

1　目的

この研究では、第 2 種スターリング数の法 n についての周期性を調べる。

1.1　用語

n 個の要素からなる集合を、どれも空集合ではない r 個の部分集合に分割する方法の総数のことを**第 2 種スターリング数** $S(n,r)$ と呼ぶ。漸化式は、$n>r>0$ のとき、

$$S(n,r) = S(n-1,r-1) + rS(n-1,r)$$

と表せる。ただし、$S(n,n)$=$S(n,1)$=1 とする。

証明

異なる n 個の集合を r 個の互いに素な空でない部分集合に分割するときに、n 個を $n-1$ 番目までと最後の n 番目だけの 2 つに分けて考える。すると、最後の n 番目が単独で入る場合とそうでない場合に場合分けができる。

最後の n 番目が単独で入る場合

$n-1$ 番目までは $(r-1)$ 個の部分集合に空でないように入っているので、入れ方は $S(n-1,r-1)$ 通りである。

最後の n 番目が単独で入らない場合

$n-1$ 番目までだけで r 個の部分集合に空でないように入っているはず。このような場合は $S(n-1,r)$ 通りあるが、最後の n 番目をどこに入れるかが r 通りに選べるので、これらの積 $rS(n-1,r)$ が入れ方の数になる。

これらの 2 つの場合の数の和 $S(n-1,r-1)+rS(n-1,r)$ が、異なる n 個の集合を r 個の互いに素な空でない部分集合に分割する方法の個数 $S(n,r)$ に等しくなる。

$n \geq r$ とするとき、n 冊の異なる本を、r 個の全く同じ本棚に空の本棚がないように入れる方法と考えるとわかりやすいと思う。

【例】$S(4,3)$ を求めてみる。4 冊の本を 3 個の全く同じ本棚に入れる方法を考えれば良いから、

① $[\{a,b\}$、$\{c\}$、$\{d\}]$
② $[\{a\}$、$\{b,c\}$、$\{d\}]$
③ $[\{a\}$、$\{b\}$、$\{c,d\}]$
④ $[\{a,c\}$、$\{b\}$、$\{d\}]$
⑤ $[\{a,d\}$、$\{b\}$、$\{c\}]$
⑥ $[\{b,d\}$、$\{a\}$、$\{c\}]$

の 6 通りに分けることができる。よって、$S(4,3)$=6 となる。

1.2　方針

漸化式に従い具体的値を表に示した。n が大きくなると値が大きくなるので、mod 2、mod 3 をした値を再び表にし、規則性を調べる。また、$S(n,r)$ の n が素数のときの規則性についても調べる。

2 方法

2.1 第2種スターリング数を求めるプログラム

```
/* 自然数Nに対してリスト [1,2,…N] を作る */
generate_e(N,E) :- gene_e_aux(E,N,[]).
gene_e_aux(L,0,L) :- !.
gene_e_aux(Result,N,L) :- N1 is N-1,!,gene_e_aux(Result,N1,[N|L]).

/* リスト同士の掛け算 */
kake_list([]=[]*[]) :- !.
kake_list([C|Z]=[A|L]*[B|M]):-
C is A*B,kake_list(Z=L*M).

/* 第2種スターリング数 */
stirling(1,[1]).
stirling(N,L) :- N>0,N1 is N-1,
stirling(N1,L1),
generate_e(N1,E),
kake_list(L2=L1*E),
append0(X=L2+[0]),
        sum_list(L=[0|L1]+X).
```

2.2 mod p を求めるプログラム

```
:-dynamic c/1.
modp(L,_,P) :- abolish(c/1),
asserta(c([])),
member(X,L),
retract1(c(A)),
X1 is X mod P,
A1=[X1|A],
asserta(c(A1)),
fail.
modp(_,M,_) :- retract1(c(M)).
```

ただし、このプログラミングだと逆に表示されてしまうので、質問するときは例えば、
?- stirling(3,L),modp(L,M,2),reverse(M,M0).
と質問すること。

3　結果

3.1　$S(n,r)$ の表

表 1: $S(n,r)$ の表 ($n \leq 29$)

$S(n,r)$	$(n \geq r \geq 1)$
$S(1,r)$	[1]
$S(2,r)$	[1,1]
$S(3,r)$	[1,3,1]
$S(4,r)$	[1,7,6,1]
$S(5,r)$	[1,15,25,10,1]
$S(6,r)$	[1,31,90,65,15,1]
$S(7,r)$	[1,63,301,350,140,21,1]
$S(8,r)$	[1,127,966,1701,1050,266,28,1]
$S(9,r)$	[1,255,3025,7770,6951,2646,462,36,1]
$S(10,r)$	[1,511,9330,34105,42525,22827,5880,750,45,1]
$S(11,r)$	[1,1023,28501,145750,246730,179487,63987,11880,1155,55,1]
$S(12,r)$	[1,2047,86526,611501,1379400,1323652,627396,159027,22275,1705,66,1]
$S(13,r)$	[1,4095,261625,2532530,7508501,9321312,5715424,1899612,359502,39325,2431,78,1]
$S(14,r)$	[1,8191,788970,10391745,40075035,63436373,49329280,20912320,5135130,752752,66066,3367,91,1]
$S(15,r)$	[1,16383,2375101,42355950,210766920,420693273,408741333,216627840,67128490,12662650,1479478,
	106470,4550,105,1]
$S(16,r)$	[1,32767,7141686,171798901,1096190550,2734926558,3281882604,2141764053,820784250,193754990,
	28936908,2757118,165620,6020,120,1]
$S(17,r)$	[1,65535,21457825,694337290,5652751651,17505749898,25708104786,20415995028,9528822303,
	2758334150,512060978,62022324,4910178,249900,7820,136,1]
$S(18,r)$	[1,131071,64439010,2798806985,28958095545,110687251039,197462483400,189036065010,
	106175395755,37112163803,8391004908,1256328866,125854638,8408778,367200,9996,153,1]
$S(19,r)$	[1,262143,193448101,11259666950,147589284710,693081601779,1492924634839,1709751003480,
	1144614626805,477297033785,129413217791,23466951300,2892439160,243577530,13916778,
	527136,12597,171,1]

表 2: $S(n,r)$ の表 $(n \leq 29)$

$S(n,r) \quad (n \geq r \geq 1)$	
$S(20,r)$	[1,524287,580606446,45232115901,749206090500,4306078895384,11143554045652,15170932662679,
	12011282644725,5917584964655, 1900842429486,411016633391,61068660380,6302524580,
	452329200,22350954,741285,15675,190,1]
$S(21,r)$	[1,1048575,1742343625,181509070050,3791262568401,26585679462804,82310957214948,
	132511015347084,123272476465204,71187132291275,26826851689001,6833042030178,
	1204909218331,149304004500,13087462580,809944464,34952799,1023435,19285,210,1]
$S(22,r)$	[1,2097151,5228079450,727778623825,19137821912055,163305339345225,602762379967440,
	1142399079991620,1241963303533920,835143799377954,366282500870286,108823356051137,
	22496861868481,3295165281331,345615943200,26046574004,1404142047,53374629,1389850,
	23485,231,1]
$S(23,r)$	[1,4194303,15686335501,2916342574750,96416888184100,998969857983405,4382641999117305,
	9741955019900400,12320068811796900,9593401297313460,4864251308951100,1672162773483930,
	401282560341390,68629175807115,8479404429331,762361127264,49916988803,2364885369,
	79781779,1859550,28336,253,1]
$S(24,r)$	[1,8388607,47063200806,11681056634501,485000783495250,6090236036084530,31677463851804540,
	82318282158320505,120622574326072500,108254081784931500,63100165695775560,
	24930204590758260,6888836057922000,1362091021641000,195820242247080,20677182465555,
	1610949936915,92484925445,3880739170,116972779,2454606,33902,276,1]
$S(25,r)$	[1,16777215,141197991025,46771289738810,2436684974110751,37026417000002430,
	227832482998716310,690223721118368580,1167921451092973005,1203163392175387500,
	802355904438462660,362262620784874680,114485073343744260,25958110360896000,
	4299394655347200,526655161695960,48063331393110,3275678594925,166218969675,6220194750,
	168519505,3200450,40250,300,1]

表 3: $S(n,r)$ の表 $(n \leq 29)$

$S(n,r) \quad (n \geq r \geq 1)$	
$S(26,r)$	[1,33554431,423610750290,187226356946265,12230196160292565,224595186974125331,
	1631853797991016600,5749622251945664950,11201516780955125625,13199555372846848005,
	10029078340998476760,5149507353856958820,1850568574253550060,477898618396288260,
	90449030191104000,12725877242482560,1343731795378830,107025546101760,6433839018750,
	290622864675,9759104355,238929405,4126200,47450,325,1]
$S(27,r)$	[1,67108863,1270865805301,749329038535350,61338207158409090,1359801318005044551,
	11647571772911241531,47628831813556336200,106563273280541795575,143197070509423605675,
	123519417123830092365,71823166587281982600,29206898819153109600,8541149231801585700,
	1834634071262848260,294063066070824960,35569317763922670,3270191625210510,
	229268487458010,12246296312250,495564056130,15015551265,333832005,5265000,
	55575,351,1]
$S(28,r)$	[1,134217727,3812664524766,2998587019946701,307440364830580800,8220146115188676396,
	82892803728383735268,392678226281361931131,1006698291338432496375,1538533978374777852325,
	1501910658871554621690,985397416171213883565,451512851236272407400,148782988064375309400,
	36060660300744309600,6539643128396047620,898741468057510350,94432767017711850,
	7626292886912700,474194413703010,22653141490980,825906183960,22693687380,460192005,
	6654375,64701,378,1]
$S(29,r)$	[1,268435455,11438127792025,11998160744311570,1540200411172850701,49628317055962639176,
	588469772213874823272,3224318613979279184316,9452962848327254398506,
	16392038075086211019625,18059551225961878690915,13326679652926121224470,
	6855064482242755179765,2534474684137526739000,689692892575539953400,
	140694950355081071520,21818248085373723570,2598531274376323650,239332331869053150,
	17110181160972900,949910385013590,40823077538100,1347860993700,33738295500,626551380,
	8336601,74907,406,1]

3.2 $S(n,r)$ (mod 2) の表

表 4:

$S(n,r)$ のリスト　$(r \leq n \leq 100)$	
$S(n,1)$	[1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,
	1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,
	1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,
	1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1]
$S(n,2)$	[1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,
	1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,
	1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,
	1,1,1,1,1,1,1,1,1,1,1,1,1,1,1]
$S(n,3)$	[1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,
	1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,
	1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,
	1,0,1,0,1,0,1,0,1,0,1,0,1,0]
$S(n,4)$	[1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,
	1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,
	1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,
	1,0,1,0,1,0,1,0,1,0,1,0,1]
$S(n,5)$	[1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,
	1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,
	1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,
	1,1,0,0,1,1,0,0,1,1,0,0]
$S(n,6)$	[1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,
	1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,
	1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,
	1,1,0,0,1,1,0,0,1,1,0]
$S(n,7)$	[1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,
	1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,
	1,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,
	1,0,0,0,1,0,0,0,1,0]

表 5:

$S(n,r)$ のリスト　$(r \leq n \leq 100)$	
$S(n,8)$	[1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,
	1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,
	1,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,
	1,0,0,0,1,0,0,0,1]
$S(n,9)$	[1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,1,1,1,1,
	0,0,0,0,1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,
	1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,1,1,1,1,
	0,0,0,0,1,1,1,1]
$S(n,10)$	[1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,1,1,1,1,
	0,0,0,0,1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,
	1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,1,1,1,1,
	0,0,0,0,1,1,1]
$S(n,11)$	[1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,1,0,1,0,
	0,0,0,0,1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,
	1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,1,0,1,0,
	0,0,0,0,1,0]
$S(n,12)$	[1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,1,0,1,0,
	0,0,0,0,1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,
	1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,1,0,1,0,
	0,0,0,0,1]
$S(n,13)$	[1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,1,1,0,0,
	0,0,0,0,1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,
	1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,1,1,0,0,
	0,0,0,0]
$S(n,14)$	[1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,1,1,0,0,
	0,0,0,0,1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,
	1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,1,1,0,0,
	0,0,0]

表 6:

$S(n,r)$ のリスト　$(r \leq n \leq 100)$	
$S(n,15)$	[1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,
	0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,
	1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0]
$S(n,16)$	[1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,
	0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,
	1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0]
$S(n,17)$	[1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1,0,0,0,0,
	0,0,0,0,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1,
	0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,1,1,1,1]
$S(n,18)$	[1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1,0,0,0,0,
	0,0,0,0,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1,
	0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,1,1,1]
$S(n,19)$	[1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,1,0,1,0,1,0,1,0,0,0,0,0,
	0,0,0,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,1,0,1,0,1,0,1,0,
	0,0,0,0,0,0,0,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,1,0]
$S(n,20)$	[1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,1,0,1,0,1,0,1,0,0,0,0,0,
	0,0,0,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,1,0,1,0,1,0,1,0,
	0,0,0,0,0,0,0,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,1]
$S(n,21)$	[1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,1,1,0,0,1,1,0,0,0,0,0,0,
	0,0,0,0,1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,1,1,0,0,1,1,0,0,
	0,0,0,0,0,0,0,0,1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0]
$S(n,22)$	[1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,1,1,0,0,1,1,0,0,0,0,0,0,
	0,0,0,0,1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,1,1,0,0,1,1,0,0,
	0,0,0,0,0,0,0,0,1,1,0,0,1,1,0,0,0,0,0,0,0,0,0]
$S(n,23)$	[1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,
	0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,
	0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0]

表 7:

$S(n,r)$ のリスト　$(r \leq n \leq 100)$	
$S(n,24)$	[1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,
	0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,
	0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0]
$S(n,25)$	[1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,
	0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0,
	0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0]
$S(n,26)$	[1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,
	0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0,
	0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0]
$S(n,27)$	[1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,
	0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,0,0,0,0,
	0,0,0,0,0,0,0,0,1,0,1,0,0,0,0,0,0,0]
$S(n,28)$	[1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,
	0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,0,0,0,0,
	0,0,0,0,0,0,0,0,1,0,1,0,0,0,0,0,0]
$S(n,29)$	[1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,
	0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,
	0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0]
$S(n,30)$	[1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,
	0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,
	0,0,0,0,0,0,0,0,1,1,0,0,0,0,0]
$S(n,31)$	[1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,
	0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,
	0,0,0,0,0,0,0,0,1,0,0,0,0,0]
$S(n,32)$	[1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,
	0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,
	0,0,0,0,0,0,0,0,1,0,0,0,0]

表 8:

$S(n,r)$ のリスト　$(r \leq n \leq 100)$	
$S(n,33)$	[1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,1,1]
$S(n,34)$	[1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,1]
$S(n,35)$	[1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,0]
$S(n,36)$	[1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1]
$S(n,37)$	[1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]
$S(n,38)$	[1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]
$S(n,39)$	[1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]
$S(n,40)$	[1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]
$S(n,41)$	[1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]
$S(n,42)$	[1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]
$S(n,43)$	[1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]

表 9:

$S(n,r)$ のリスト　$(r \leq n \leq 100)$	
$S(n,44)$	[1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0]
$S(n,45)$	[1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0]
$S(n,46)$	[1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0]
$S(n,47)$	[1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0]
$S(n,48)$	[1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0]
$S(n,49)$	[1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0]
$S(n,50)$	[1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0]
$S(n,51)$	[1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0]
$S(n,52)$	[1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0]
$S(n,53)$	[1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,0,0,1,1,0,0,0,0,0,0,0,0,0]
$S(n,54)$	[1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,0,0,1,1,0,0,0,0,0,0,0,0]
$S(n,55)$	[1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,0,0,0,1,0,0,0,0,0,0,0,0]
$S(n,56)$	[1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,0,0,0,1,0,0,0,0,0,0,0]
$S(n,57)$	[1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,1,1,0,0,0,0,0,0,0]

表 10:

$S(n,r)$ のリスト　$(r \leq n \leq 100)$	
$S(n,58)$	[1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,1,1,0,0,0,0,0,0,0]
$S(n,59)$	[1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,0,1,0,0,0,0,0,0,0]
$S(n,60)$	[1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,0,1,0,0,0,0,0,0]
$S(n,61)$	[1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,0,0,0,0,0,0]
$S(n,62)$	[1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,1,0,0,0,0,0]
$S(n,63)$	[1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,0,0,0,0,0]
$S(n,64)$	[1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
	1,0,0,0,0]
$\vdots$	……

3.3　$S(n,r)$ (mod 3) の表

表 11:

$S(n,r)$ のリスト　$(r \leq n \leq 100)$	
$S(n,1)$	[1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,
	1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,
	1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,
	1,1,1,1,1,1,1,1,1,1]
$S(n,2)$	[1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,
	1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,
	1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,
	1,0,1,0,1,0,1,0,1]
$S(n,3)$	[1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,
	1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,
	1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,
	1,0,1,0,1,0,1,0]
$S(n,4)$	[1,1,2,2,0,0,1,1,2,2,0,0,1,1,2,2,0,0,1,1,2,2,0,0,1,1,2,2,0,0,
	1,1,2,2,0,0,1,1,2,2,0,0,1,1,2,2,0,0,1,1,2,2,0,0,1,1,2,2,0,0,
	1,1,2,2,0,0,1,1,2,2,0,0,1,1,2,2,0,0,1,1,2,2,0,0,1,1,2,2,0,0,
	1,1,2,2,0,0,1]
$S(n,5)$	[1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,
	1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,
	1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,
	1,0,2,0,0,0]
$S(n,6)$	[1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,
	1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,
	1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,
	1,0,2,0,0]
$S(n,7)$	[1,1,0,0,0,0,1,1,0,0,0,0,1,1,0,0,0,0,1,1,0,0,0,0,1,1,0,0,0,0,
	1,1,0,0,0,0,1,1,0,0,0,0,1,1,0,0,0,0,1,1,0,0,0,0,1,1,0,0,0,0,
	1,1,0,0,0,0,1,1,0,0,0,0,1,1,0,0,0,0,1,1,0,0,0,0,1,1,0,0,0,0,
	1,1,0,0]

表 12:

$S(n,r)$ のリスト　($r \leq n \leq 100$)	
$S(n,8)$	[1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,
	1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,
	1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,
	1,0,0]
$S(n,9)$	[1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,
	1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,
	1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,
	1,0]
$S(n,10)$	[1,1,1,1,1,1,2,2,2,2,2,2,0,0,0,0,0,0,1,1,1,1,1,1,2,2,2,2,2,2,
	0,0,0,0,0,0,1,1,1,1,1,1,2,2,2,2,2,2,0,0,0,0,0,0,1,1,1,1,1,1,
	2,2,2,2,2,2,0,0,0,0,0,0,1,1,1,1,1,1,2,2,2,2,2,2,0,0,0,0,0,0,
	1]
$S(n,11)$	[1,0,1,0,1,0,2,0,2,0,2,0,0,0,0,0,0,0,1,0,1,0,1,0,2,0,2,0,2,0,
	0,0,0,0,0,0,1,0,1,0,1,0,2,0,2,0,2,0,0,0,0,0,0,0,1,0,1,0,1,0,
	2,0,2,0,2,0,0,0,0,0,0,0,1,0,1,0,1,0,2,0,2,0,2,0,0,0,0,0,0,0]
$S(n,12)$	[1,0,1,0,1,0,2,0,2,0,2,0,0,0,0,0,0,0,1,0,1,0,1,0,2,0,2,0,2,0,
	0,0,0,0,0,0,1,0,1,0,1,0,2,0,2,0,2,0,0,0,0,0,0,0,1,0,1,0,1,0,
	2,0,2,0,2,0,0,0,0,0,0,0,1,0,1,0,1,0,2,0,2,0,2,0,0,0,0,0,0,0]
$S(n,13)$	[1,1,2,2,0,0,2,2,1,1,0,0,0,0,0,0,0,0,1,1,2,2,0,0,2,2,1,1,0,0,
	0,0,0,0,0,0,1,1,2,2,0,0,2,2,1,1,0,0,0,0,0,0,0,0,1,1,2,2,0,0,
	2,2,1,1,0,0,0,0,0,0,0,0,1,1,2,2,0,0,2,2,1,1,0,0,0,0,0,0]
$S(n,14)$	[1,0,2,0,0,0,2,0,1,0,0,0,0,0,0,0,0,0,1,0,2,0,0,0,2,0,1,0,0,0,
	0,0,0,0,0,0,1,0,2,0,0,0,2,0,1,0,0,0,0,0,0,0,0,0,1,0,2,0,0,0,
	2,0,1,0,0,0,0,0,0,0,0,0,1,0,2,0,0,0,2,0,1,0,0,0,0,0,0]
$S(n,15)$	[1,0,2,0,0,0,2,0,1,0,0,0,0,0,0,0,0,0,1,0,2,0,0,0,2,0,1,0,0,0,
	0,0,0,0,0,0,1,0,2,0,0,0,2,0,1,0,0,0,0,0,0,0,0,0,1,0,2,0,0,0,
	2,0,1,0,0,0,0,0,0,0,0,0,1,0,2,0,0,0,2,0,1,0,0,0,0,0,0]

表 13:

$S(n,r)$ のリスト　$(r \leq n \leq 100)$	
$S(n,16)$	[1,1,0,0,0,0,2,2,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,2,2,0,0,0,0,
	0,0,0,0,0,0,1,1,0,0,0,0,2,2,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,
	2,2,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,2,2,0,0,0,0,0]
$S(n,17)$	[1,0,0,0,0,0,2,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,2,0,0,0,0,0,
	0,0,0,0,0,0,1,0,0,0,0,0,2,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,
	2,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,2,0,0,0,0,0]
$S(n,18)$	[1,0,0,0,0,0,2,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,2,0,0,0,0,0,
	0,0,0,0,0,0,1,0,0,0,0,0,2,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,
	2,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,2,0,0,0,0]
$S(n,19)$	[1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,1,1,0,0,0,0,0,0,
	0,0,0,0,0,0,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,1,1,
	0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,1,1,0,0,0,0]
$S(n,20)$	[1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,1,0,0,0,0,0,0,0,
	0,0,0,0,0,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,1,0,
	0,0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,1,0,0,0,0]
$S(n,21)$	[1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,1,0,0,0,0,0,0,0,
	0,0,0,0,0,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,1,0,
	0,0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,1,0,0,0]
$S(n,22)$	[1,1,2,2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,2,2,0,0,0,0,0,0,0,0,
	0,0,0,0,0,0,1,1,2,2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,2,2,0,0,
	0,0,0,0,0,0,0,0,0,0,0,0,1,1,2,2,0,0,0]
$S(n,23)$	[1,0,2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,2,0,0,0,0,0,0,0,0,0,
	0,0,0,0,0,0,1,0,2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,2,0,0,0,
	0,0,0,0,0,0,0,0,0,0,0,0,1,0,2,0,0,0]
$S(n,24)$	[1,0,2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,2,0,0,0,0,0,0,0,0,0,
	0,0,0,0,0,0,1,0,2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,2,0,0,0,
	0,0,0,0,0,0,0,0,0,0,0,0,1,0,2,0,0]

表 14:

$S(n,r)$ のリスト　$(r \leq n \leq 100)$	
$S(n,25)$	[1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,
	0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,
	0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0]
$S(n,26)$	[1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,
	0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,
	0,0,0,0,0,0,0,0,0,0,0,0,1,0,0]
$S(n,27)$	[1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,
	0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,
	0,0,0,0,0,0,0,0,0,0,0,0,1,0]
$S(n,28)$	[1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,2,2,2,2,
	2,2,2,2,2,2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,1,1,
	1,1,1,1,1,1,1,1,1,1,1,1,2]
$S(n,29)$	[1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,2,0,2,0,2,0,2,0,2,0,2,0,
	2,0,2,0,2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,1,0,
	1,0,1,0,1,0,1,0,1,0,1,0]
$S(n,30)$	[1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,2,0,2,0,2,0,2,0,2,0,2,0,
	2,0,2,0,2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,1,0,
	1,0,1,0,1,0,1,0,1,0,1]
$S(n,31)$	[1,1,2,2,0,0,1,1,2,2,0,0,1,1,2,2,0,0,2,2,1,1,0,0,2,2,1,1,0,0,
	2,2,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,2,2,0,0,
	1,1,2,2,0,0,1,1,2,2]
$S(n,32)$	[1,0,2,0,0,0,1,0,2,0,0,0,1,0,2,0,0,0,2,0,1,0,0,0,2,0,1,0,0,0,
	2,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,2,0,0,0,
	1,0,2,0,0,0,1,0,2]
$\vdots$	$\cdots\cdots$

4　考察

4.1　$S(n,r)$ (mod 2) においての予想

r を固定して、$S(n,r)$ (mod 2) を数列として考える。ただし、$n \geq r$ なので、

初項は、$S(r,r)$ (mod 2)=1
第 2 項は、$S(r+1,r)$ (mod 2)
第 3 項は、$S(r+2,r)$ (mod 2)
$\vdots$

こうしてできた数列を、$\{A(r)_n\}_{n=1}^{\infty}$ と書くことにする。これまでの結果によって次のことが予想できる。

- $\{A(1)_n\}$ と $\{A(2)_n\}$ は定数列 $\{1,1,1,1,\cdots\}$ で周期 1。
- $\{A(3)_n\}$ と $\{A(4)_n\}$ は定数列 $\{1,0,1,0,\cdots\}$ となり 1,0 が繰り返され周期 2。
- $\{A(5)_n\}$ と $\{A(6)_n\}$ は定数列 $\{1,1,0,0,1,1,0,0,\cdots\}$ となり 1,1,0,0 が繰り返され周期 4。
- $\{A(7)_n\}$ と $\{A(8)_n\}$ は定数列 $\{1,0,0,0,1,0,0,0,\cdots\}$ となり 1,0,0,0 が繰り返され周期 4。
- $\{A(9)_n\}$ と $\{A(10)_n\}$ は定数列 $\{1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,\cdots\}$ となり 1,1,1,1,0,0,0,0 が繰り返され周期 8。
- $\{A(11)_n\}$ と $\{A(12)_n\}$ は定数列 $\{1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,\cdots\}$ となり 1,0,1,0,0,0,0,0 が繰り返され周期 8。
- $\{A(13)_n\}$ と $\{A(14)_n\}$ は定数列 $\{1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,\cdots\}$ となり 1,1,0,0,0,0,0,0 が繰り返され周期 8。
- $\{A(15)_n\}$ と $\{A(16)_n\}$ は定数列 $\{1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,\cdots\}$ となり 1,0,0,0,0,0,0,0 が繰り返され周期 8。
- $\{A(17)_n\}$ と $\{A(18)_n\}$ は定数列 $\{1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0 が繰り返され周期 16。
- $\{A(19)_n\}$ と $\{A(20)_n\}$ は定数列 $\{1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0 が繰り返され周期 16。
- $\{A(21)_n\}$ と $\{A(22)_n\}$ は定数列 $\{1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 16。
- $\{A(23)_n\}$ と $\{A(24)_n\}$ は定数列 $\{1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 16。
- $\{A(25)_n\}$ と $\{A(26)_n\}$ は定数列 $\{1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 16。

- $\{A(27)_n\}$ と $\{A(28)_n\}$ は定数列 $\{1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 16。

- $\{A(29)_n\}$ と $\{A(30)_n\}$ は定数列 $\{1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 16。

- $\{A(31)_n\}$ と $\{A(32)_n\}$ は定数列 $\{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 16。

- $\{A(33)_n\}$ と $\{A(34)_n\}$ は定数列 $\{1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 32。

- $\{A(35)_n\}$ と $\{A(36)_n\}$ は定数列 $\{1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 32。

- $\{A(37)_n\}$ と $\{A(38)_n\}$ は定数列 $\{1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 32。

- $\{A(39)_n\}$ と $\{A(40)_n\}$ は定数列 $\{1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 32。

- $\{A(41)_n\}$ と $\{A(42)_n\}$ は定数列 $\{1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 32。

- $\{A(43)_n\}$ と $\{A(44)_n\}$ は定数列 $\{1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 32。

- $\{A(45)_n\}$ と $\{A(46)_n\}$ は定数列 $\{1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 32。

- $\{A(47)_n\}$ と $\{A(48)_n\}$ は定数列 $\{1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 32。

- $\{A(49)_n\}$ と $\{A(50)_n\}$ は定数列 $\{1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 32。

- $\{A(51)_n\}$ と $\{A(52)_n\}$ は定数列 $\{1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 32。

- $\{A(53)_n\}$ と $\{A(54)_n\}$ は定数列 $\{1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 32。

- $\{A(55)_n\}$ と $\{A(56)_n\}$ は定数列 $\{1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 32。

- $\{A(57)_n\}$ と $\{A(58)_n\}$ は定数列 $\{1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 32。

- $\{A(59)_n\}$ と $\{A(60)_n\}$ は定数列 $\{1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 32。

- $\{A(61)_n\}$ と $\{A(62)_n\}$ は定数列 $\{1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 32。

- $\{A(63)_n\}$ と $\{A(64)_n\}$ は定数列 $\{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 32。

$$\vdots$$

次に、周期が 32 の時まで確かめる。

4.2 $S(n,r)$ (mod 2) においての予想　証明

手計算をして予想が正しいか確かめる。
まず、基準になる行に 1 から順に数字をふっていく。例えば、7 に注目してみると、上の行の 3 と基準になる行の 2 を掛け合わせて、前の行の左の 1 を足し合わせることで表せる (すなわち、漸化式 $S(n,r) = S(n-1,r-1) + rS(n-1,r)$ に従って求めていることがわかる)。この計算を上から順にしていくと、先程求めた $S(n,r)$ の表の結果と合致する。(表 15)

表 15:

1	2	3	4	5	6	···
1						
1	1					
1	3	1				
1	$\boxed{7}$	6	1			
1	15	25	10	1		
1	31	90	65	15	1	

$$\boxed{7} = 2 \times 3 + 1$$

基準になる行を mod 2 して、上と同様に計算すると、

表 16:

1	0	1	0	1	0	···
1						
1	1					
1	1	1				
1	1	0	1			
1	1	1	0	1		
1	1	0	1	1	1	

以下、確かめる。

注 : 周期性を確かめることが目的なので、空欄の所は"**0**"を補って考える。

周期が 1 の時は省略する。

表 17: 周期が 2 の時

1	0	1	0
1			
1	1		
1	1	1	
1	1	0	1
1	1	1	0
1	1	0	1
1	1	1	0
1	1	0	1

3 列目、4 列目において周期が 2 で繰り返されるので、予想は正しい。

表 18: 周期が 4 の時

1	0	1	0	1	0	1	0
1							
1	1						
1	1	1					
1	1	0	1				
1	1	1	0	1	0	0	0
1	1	0	1	1	1	0	0
1	1	1	0	0	1	1	0
1	1	0	1	0	0	0	1
1	1	1	0	1	0	0	0
1	1	0	1	1	1	0	0
1	1	1	0	0	1	1	0
1	1	0	1	0	0	0	1
1	1	1	0	1	0	0	0

5 列目から 8 列目において周期が 4 で繰り返されるので、予想は正しい。

表 19: 周期が 8 の時

1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
1															
1	1														
1	1	1													
1	1	0	1												
1	1	1	0	1											
1	1	0	1	1	1										
1	1	1	0	0	1	1									
1	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0
1	1	1	0	1	0	0	0	1	0	0	0	0	0	0	0
1	1	0	1	1	1	0	0	1	1	0	0	0	0	0	0
1	1	1	0	0	1	1	0	1	1	1	0	0	0	0	0
1	1	0	1	0	0	0	1	1	1	0	1	0	0	0	0
1	1	1	0	1	0	0	0	0	1	1	0	1	0	0	0
1	1	0	1	1	1	0	0	0	0	0	1	1	1	0	0
1	1	1	0	0	1	1	0	0	0	0	0	0	1	1	0
1	1	0	1	0	0	0	1	0	0	0	0	0	0	0	1
1	1	1	0	1	0	0	0	1	0	0	0	0	0	0	0
1	1	0	1	1	1	0	0	1	1	0	0	0	0	0	0
1	1	1	0	0	1	1	0	1	1	1	0	0	0	0	0
1	1	0	1	0	0	0	1	1	1	0	1	0	0	0	0
1	1	1	0	1	0	0	0	0	1	1	0	1	0	0	0
1	1	0	1	1	1	0	0	0	0	0	1	1	1	0	0
1	1	1	0	0	1	1	0	0	0	0	0	0	1	1	0
1	1	0	1	0	0	0	1	0	0	0	0	0	0	0	1

9 列目から 16 列目において周期が 8 で繰り返されるので、予想は正しい。

表 20: 周期が 16 の時—その 1

1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0
1
1 1
1 1 1
1 1 0 1
1 1 1 0 1
1 1 0 1 1 1
1 1 1 0 0 1 1
1 1 0 1 0 0 0 1
1 1 1 0 1 0 0 0 1
1 1 0 1 1 1 0 0 1 1
1 1 1 0 0 1 1 0 1 1 1
1 1 0 1 0 0 0 1 1 1 0 1
1 1 1 0 1 0 0 0 0 1 1 0 1
1 1 0 1 1 1 0 0 0 0 0 1 1 1
1 1 1 0 0 1 1 0 0 0 0 0 0 1 1
1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1
1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1
1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 1 1
1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 1 1 1
1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 1 0 0 1 0 1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0
1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0
1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0
1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0
1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 1 1 1 0 0 1 1 0 0 0 0 0 0
1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 1 1 0 1 1 1 0 0 0 0 0
1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 0
1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0
1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0
1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0
1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0

表 21: 周期が 16 の時—その 2

1	1	1	0	1	0	0	1	0	1	1	0	1	0	0	0	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0
1	1	0	1	1	1	0	0	0	0	0	1	1	1	0	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0
1	1	1	0	0	1	1	0	0	0	0	0	0	1	1	0	1	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	1	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0
1	1	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	1	0	0	0	0	0	0	0
1	1	0	1	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1	1	0	0	0	0	0	0
1	1	1	0	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	1	0	0	0	0	0
1	1	0	1	0	0	0	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	0
1	1	1	0	1	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0
1	1	0	1	1	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0
1	1	1	0	0	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
1	1	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1	1	1	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	1	1	1	0	0	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	0	0	1	1	0	1	1	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	1	1	1	0	1	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0

17 列目から 32 列目において周期が 16 で繰り返されるので、予想は正しい。

表 22: 周期が 32 の時—その 1

1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0
1
1 1
1 1 1
1 1 0 1
1 1 1 0 1
1 1 0 1 1 1
1 1 1 0 0 1 1
1 1 0 1 0 0 0 1
1 1 1 0 1 0 0 0 1
1 1 0 1 1 1 0 0 1 1
1 1 1 0 0 1 1 0 1 1 1
1 1 0 1 0 0 0 1 1 1 0 1
1 1 1 0 1 0 0 0 0 1 1 0 1
1 1 0 1 1 1 0 0 0 0 0 1 1 1
1 1 1 0 0 1 1 0 0 0 0 0 0 1 1
1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1
1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1
1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 1 1
1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 1 1 1
1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 1 1 0 1
1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 1 1 1 0 1
1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 1 1 0 1 1 1
1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 1 1 1 0 0 1 1
1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 1
1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0 1
1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 1 1 1 0 0 1 1
1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 1 1 0 1 1 1
1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 1
1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1
1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1
1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

表 23: 周期が 32 の時—その 2

1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0
1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0
1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0
1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0
1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 1 1 0 0 0 0 0 0
1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 1 1 0 0 0 0 0
1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 0
1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0
1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0
1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0
1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1

表 24: 周期が 32 の時—その 3

1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0	1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0	1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0	1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0	1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0	1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0	1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0	1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 1 1 1 0 0 1 1 0 0 0 0 0 0	1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 1 1 0 1 1 1 0 0 0 0 0	1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 0	1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0	1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0	1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0	1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1	1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	0 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0	0 0 0 1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0
1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0
1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0
1 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0 1 0 0 0 0 0 0 0
1 1 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 1 1 1 0 0 1 1 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 1 1 0 0 0 0 0 0
1 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 1 1 0 1 1 1 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 1 1 0 0 0 0 0
1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 0
1 1 1 0 1 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0
1 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0
1 1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0
1 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1

33 列目から 64 列目において周期が 32 で繰り返されるので、予想は正しい。

4.3 $S(n,r)$ (mod 3) においての予想

r を固定して、$S(n,r)$ (mod 3) を数列として考える。$S(n,r)$ (mod 2) と同様に考えると、これまでの結果から次のことが予想できる。

- $\{A(1)_n\}$ は定数列 $\{1,1,1,1,\cdots\}$ で周期 1。
- $\{A(2)_n\}$ と $\{A(3)_n\}$ は定数列 $\{1,0,1,0,\cdots\}$ となり 1,0 が繰り返され周期 2。
- $\{A(4)_n\}$ は定数列 $\{1,1,2,2,0,0,1,1,2,2,0,0,\cdots\}$ となり 1,1,2,2,0,0 が繰り返され周期 6。
- $\{A(5)_n\}$ と $\{A(6)_n\}$ は定数列 $\{1,0,2,0,0,0,1,0,2,0,0,0,\cdots\}$ となり 1,0,2,0,0,0 が繰り返され周期 6。
- $\{A(7)_n\}$ は定数列 $\{1,1,0,0,0,0,1,1,0,0,0,0,\cdots\}$ となり 1,1,0,0,0,0 が繰り返され周期 6。
- $\{A(8)_n\}$ と $\{A(9)_n\}$ は定数列 $\{1,0,0,0,0,0,1,0,0,0,0,0,\cdots\}$ となり 1,0,0,0,0,0 が繰り返され周期 6。
- $\{A(10)_n\}$ は定数列 $\{1,1,1,1,1,1,2,2,2,2,2,2,0,0,0,0,0,0,\cdots\}$ となり 1,1,1,1,1,1,2,2,2,2,2,2,0,0,0,0,0,0 が繰り返され周期 18。
- $\{A(11)_n\}$ と $\{A(12)_n\}$ は定数列 $\{1,0,1,0,1,0,2,0,2,0,2,0,0,0,0,0,0,0,\cdots\}$ となり 1,0,1,0,1,0,2,0,2,0,2,0,0,0,0,0,0,0 が繰り返され周期 18。
- $\{A(13)_n\}$ は定数列 $\{1,1,2,2,0,0,2,2,1,1,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,1,2,2,0,0,2,2,1,1,0,0,0,0,0,0,0,0 が繰り返され周期 18。
- $\{A(14)_n\}$ と $\{A(15)_n\}$ は定数列 $\{1,0,2,0,0,0,2,0,1,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,0,2,0,0,0,2,0,1,0,0,0,0,0,0,0,0,0 が繰り返され周期 18。
- $\{A(16)_n\}$ は定数列 $\{1,1,0,0,0,0,2,2,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,1,1,1,1,1,2,2,2,2,2,2,0,0,0,0,0,0 が繰り返され周期 18。
- $\{A(17)_n\}$ と $\{A(18)_n\}$ は定数列 $\{1,0,0,0,0,0,2,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,0,0,0,0,0,2,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 18。
- $\{A(19)_n\}$ は定数列 $\{1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 18。
- $\{A(20)_n\}$ と $\{A(21)_n\}$ は定数列 $\{1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり 1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 18。

- $\{A(22)_n\}$ は定数列 $\{1,1,2,2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり
  1,1,2,2,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 18。

- $\{A(23)_n\}$ と $\{A(24)_n\}$ は定数列 $\{1,0,2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり
  1,0,2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 18。

- $\{A(25)_n\}$ は定数列 $\{1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり
  1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 18。

- $\{A(26)_n\}$ と $\{A(27)_n\}$ は定数列 $\{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\cdots\}$ となり
  1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 が繰り返され周期 18。

$$\vdots$$

次に、周期が 18 の時まで確かめる。

4.4 $S(n,r)$ (mod 3) においての予想　証明

$S(n,r)$ (mod 2) と同様に確かめる。ただし、周期が 1 の時は省略する。

表 25: 周期が 2 の時

1	2	0
1		
1	1	
1	0	1
1	1	0
1	0	1
1	1	0
1	0	1

2 行目、3 行目において周期が 2 で繰り返されるので、予想は正しい。

表 26: 周期が 6 の時

1	2	0	1	2	0	1	2	0
1								
1	1							
1	0	1						
1	1	0	1					
1	0	1	1	1	0	0	0	0
1	1	0	2	0	1	0	0	0
1	0	1	2	2	0	1	0	0
1	1	0	0	0	2	1	1	0
1	0	1	0	0	0	0	0	1
1	1	0	1	0	0	0	0	0
1	0	1	1	1	0	0	0	0
1	1	0	2	0	1	0	0	0
1	0	1	2	2	0	1	0	0
1	1	0	0	0	2	1	1	0
1	0	1	0	0	0	0	0	1
1	1	0	1	0	0	0	0	0
1	0	1	1	1	0	0	0	0

4 行目から 9 行目において周期が 6 で繰り返されるので、予想は正しい。

表 27: 周期が 18 の時—その 1

1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0
1																										
1	1																									
1	0	1																								
1	1	0	1																							
1	0	1	1	1																						
1	1	0	2	0	1																					
1	0	1	2	2	0	1																				
1	1	0	0	0	2	1	1																			
1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	2	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	2	0	1	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	2	1	1	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	1	1	1	0	2	0	1	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	2	0	1	2	2	0	1	0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	2	2	0	0	0	2	1	1	0	0	0	0	0	0	0	0	0	0
1	1	0	2	0	1	0	0	0	2	0	2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
1	0	1	2	2	0	1	0	0	2	2	0	2	0	0	0	0	0	1	0	0	0	0	0	0	0	0
1	1	0	0	0	2	1	1	0	2	0	2	2	2	0	0	0	0	1	1	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	1	2	2	0	1	0	2	0	0	0	1	0	1	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0	0	2	1	1	0	2	0	0	1	1	0	1	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0	0	0	0	0	1	2	2	0	1	0	1	1	1	0	0	0	0
1	1	0	2	0	1	0	0	0	0	0	0	0	0	0	0	0	2	1	1	0	2	0	1	0	0	0
1	0	1	2	2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	2	0	1	0	0
1	1	0	0	0	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	1	0
1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

表 28: 周期が 18 の時—その 2

1	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	2	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	2	0	1	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	2	1	1	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	1	1	1	0	2	0	1	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	2	0	1	2	2	0	1	0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	2	2	0	0	0	2	1	1	0	0	0	0	0	0	0	0	0	0
1	1	0	2	0	1	0	0	0	2	0	2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
1	0	1	2	2	0	1	0	0	2	2	0	2	0	0	0	0	0	1	0	0	0	0	0	0	0	0
1	1	0	0	0	2	1	1	0	2	0	2	2	2	0	0	0	0	1	1	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	1	2	2	0	1	0	2	0	0	0	1	0	1	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0	0	2	1	1	0	2	0	0	1	1	0	1	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0	0	0	0	0	1	2	2	0	1	0	1	1	1	0	0	0	0
1	1	0	2	0	1	0	0	0	0	0	0	0	0	0	0	0	2	1	1	0	2	0	1	0	0	0
1	0	1	2	2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	2	0	1	0	0
1	1	0	0	0	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	1	0
1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

10 行目から 27 行目において周期が 18 で繰り返されるので、予想は正しい。

4.5　周期のつくる数列

$S(n,r)\ (\mathrm{mod}\ 2)$、$S(n,r)\ (\mathrm{mod}\ 3)$ で求めた周期をもとに、周期のつくる数列に注目してみる。

表 29: $S(n,r)\ (\mathrm{mod}\ 2)$ の周期のつくる数列—その 1

周期	パターン
1	$\{1\}$
2	$\{1,0\}$
4	$\{1,1,0,0\}$
	$\{1,0,0,0\}$
8	$\{1,1,1,1,0,0,0,0\}$
	$\{1,0,1,0,0,0,0,0\}$
	$\{1,1,0,0,0,0,0,0\}$
	$\{1,0,0,0,0,0,0,0\}$
16	$\{1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0\}$
	$\{1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0\}$
	$\{1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$

表 30: $S(n,r) \pmod 2$ の周期のつくる数列—その 2

周期	パターン
32	$\{1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$

表 31: $S(n,r) \pmod 3$ の周期のつくる数列

周期	パターン
1	$\{1\}$
2	$\{1,0\}$
6	$\{1,1,2,2,0,0\}$
	$\{1,0,2,0,0,0\}$
	$\{1,1,0,0,0,0\}$
	$\{1,0,0,0,0,0\}$
18	$\{1,1,1,1,1,1,2,2,2,2,2,2,0,0,0,0,0,0\}$
	$\{1,0,1,0,1,0,2,0,2,0,2,0,0,0,0,0,0,0\}$
	$\{1,1,2,2,0,0,2,2,1,1,0,0,0,0,0,0,0,0\}$
	$\{1,0,2,0,0,0,2,0,1,0,0,0,0,0,0,0,0,0\}$
	$\{1,1,1,1,1,1,2,2,2,2,2,2,0,0,0,0,0,0\}$
	$\{1,0,0,0,0,0,2,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,1,2,2,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,0,2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$
	$\{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$

推測として、

- $S(n,r) \pmod 2$ の周期だけを見てみると、

  $$1,2,4,8,16,32,\cdots$$

  初項が 1、公比が 2 の等比数列 ($a_n = 2^{n-1}$) で周期が続いていくと予想できる。

- また、$S(n,r) \pmod 3$ の周期だけを見てみると、

  $$1,2,6,18,\cdots$$

  初項が 2、公比が 3 の等比数列 ($a_n = 2 \cdot 3^{n-1}$) で周期が続いていくと予想できる。
  (ただし、はじめの 1 は除く)

4.6 $S(n,r)$ の n が素数のときの予想

表 1 から表 3 に注目してみる。$\boxed{n = \text{素数 } p \text{ とすると、} S(p,r) \text{ は } 1<r<p \text{ のとき } p \text{ の倍数である}}$ と予想できる。

4.7 $S(n,r)$ の n が素数のときの予想　証明

$r = 2, 3, 4, 5$ のとき、

$$
\begin{array}{ll}
S(p,2) & 2<p \\
S(p,3) & 3<p \\
S(p,4) & 4<p \\
S(p,5) & 5<p
\end{array}
$$

の証明をする。

証明する際に、以下の小定理を利用する。

フェルマーの小定理

p が素数、a が p で割れないなら、

$$a^{p-1} - 1 = kp$$

と書ける。

また、第 2 種スターリング数の基本的公式も利用する。

$$S(n,r) = s_{n,r}/r! = \left(\sum_{k=0}^{r}(-1)^k \ {}_rC_k(r-k)^n\right)/r!$$

- $S(p,2)$ の時の証明

表 32:

n＼r	2 列目
1 行目	
2 行目	1
3 行目	3
4 行目	7
5 行目	15
6 行目	31
7 行目	63
8 行目	127
9 行目	255
10 行目	511
11 行目	1023
12 行目	2047
13 行目	4095
$\vdots$	$\vdots$

$$S(p,2) = (2^p - 2)/2! = 2^{p-1} - 1$$

(フェルマーの小定理を利用して、$2^{p-1} = 1 + kp$ と変形できる)

$$= 1 + kp - 1 = kp$$

$$= p \text{ の倍数}$$

[証明終]

- $S(p,3)$ の時の証明

表 33:

n＼r	3 列目
1 行目	
2 行目	
3 行目	1
4 行目	6
5 行目	25
6 行目	90
7 行目	301
8 行目	966
9 行目	3025
10 行目	9330
11 行目	28501
12 行目	86526
13 行目	261625
$\vdots$	$\vdots$

$$S(p,3) = \left(3^p - 3 \times 2^p + 3\right)/3!$$
$$= \frac{1}{2} \cdot 3^{p-1} - 2^{p-1} + \frac{1}{2}$$

(フェルマーの小定理を利用して、$3^{p-1} = 1 + kp$、$2^{p-1} = 1 + lp$ と変形できる)

$$= \frac{1}{2}(1 + kp) - (1 + lp) + \frac{1}{2}$$
$$= \left(\frac{1}{2}k - l\right)p + \frac{1}{2} - 1 + \frac{1}{2}$$
$$= \frac{pk}{2} - pl$$

($\frac{pk}{2}$ は整数だから、$k = 2k!$ と書ける)

$$= (k! - l)p$$
$$= p \text{ の倍数}$$

[証明終]

- $S(p,4)$ の時の証明

表 34:

n＼r	4 列目
1 行目	
2 行目	
3 行目	
4 行目	1
5 行目	10
6 行目	65
7 行目	350
8 行目	1701
9 行目	7770
10 行目	34105
11 行目	145750
12 行目	611501
13 行目	2532530
$\vdots$	$\vdots$

$$
\begin{aligned}
S(p,4) &= \left(4^p - 4\cdot 3^p + 6\cdot 2^p - 4\right)/4! \\
&= \frac{1}{6}\cdot 4^{p-1} - \frac{1}{2}\cdot 3^{p-1} + \frac{1}{2}\cdot 2^{p-1} - \frac{1}{6}
\end{aligned}
$$

(フェルマーの小定理を利用して、$4^{p-1} = 1 + kp$、$3^{p-1} = 1 + lp$、$2^{p-1} = 1 + mp$ と変形できる)

$$
\begin{aligned}
&= \frac{1}{6}(1+kp) - \frac{1}{2}(1+lp) + \frac{1}{2}(1+mp) - \frac{1}{6} \\
&= \left(\frac{1}{6}k - \frac{1}{2}l + \frac{1}{2}m\right)p + \frac{1}{6} - \frac{1}{2} + \frac{1}{2} - \frac{1}{6} \\
&= \left(\frac{1}{6}k - \frac{1}{2}l + \frac{1}{2}m\right)p \\
&= \frac{pk}{6} - \frac{pl}{2} + \frac{pm}{2}
\end{aligned}
$$

($\frac{pk}{6}$、$\frac{pl}{2}$、$\frac{pm}{2}$ は整数だから、$k = 6k!$、$l = 2l!$、$m = 2m!$ と書ける)

$$
\begin{aligned}
&= (k! - l! + m!)p \\
&= p \text{ の倍数}
\end{aligned}
$$

[証明終]

- $S(p,5)$ の時の証明

表 35:

n＼r	5 列目
1 行目	
2 行目	
3 行目	
4 行目	
5 行目	1
6 行目	15
7 行目	140
8 行目	1050
9 行目	6951
10 行目	42525
11 行目	246730
12 行目	1379400
13 行目	7508501
$\vdots$	$\vdots$

$$\begin{aligned} S(p,5) &= \left(5^p - 5\cdot 4^p + 10\cdot 3^p - 10\cdot 2^p + 5\right)/5! \\ &= \frac{1}{24}\cdot 5^{p-1} - \frac{1}{6}\cdot 4^{p-1} + \frac{1}{4}\cdot 3^{p-1} - \frac{1}{6}\cdot 2^{p-1} + \frac{1}{24} \end{aligned}$$

(フェルマーの小定理を利用して、

$5^{p-1} = 1 + kp$、$4^{p-1} = 1 + lp$、$3^{p-1} = 1 + mp$、$2^{p-1} = 1 + op$ と変形できる)

$$\begin{aligned} &= \frac{1}{24}(1+kn) - \frac{1}{6}(1+lp) + \frac{1}{4}(1+mp) - \frac{1}{6}(1+op) + \frac{1}{24} \\ &= \left(\frac{1}{24}k - \frac{1}{6}l + \frac{1}{4}m - \frac{1}{6}o\right)p + \frac{1}{24} - \frac{1}{6} + \frac{1}{4} - \frac{1}{6} + \frac{1}{24} \\ &= \left(\frac{1}{24}k - \frac{1}{6}l + \frac{1}{4}m - \frac{1}{6}o\right)p \\ &= \frac{pk}{24} - \frac{pl}{6} + \frac{pm}{4} - \frac{po}{6} \end{aligned}$$

($\frac{pk}{24}$、$\frac{pl}{6}$、$\frac{pm}{4}$、$\frac{po}{6}$ は整数だから、$k = 24k!$、$l = 6l!$、$m = 4m!$、$o = 6o!$ と書ける)

$$= (k! - l! + m! - o!)p$$

$$= p \text{ の倍数}$$

[証明終]

4.8 課題

- 一つ一つ手計算で確かめたので、$S(n,r) \pmod 2$ は周期が 32 の時まで、$S(n,r) \pmod 3$ は周期が 18 の時まで、また $S(n,r)$ の n が素数のときの規則性も $r=5$ までしか示せなかった。r が無限に大きくなると成立するかどうかは未解決。

- 周期のつくる数列のパターンが時間が足りなく、詳しく考察することができなかった。

5 感想

前期はプログラミングをよく理解していなかったので、毎回来るのが憂鬱でしたが (笑)、後期は飯高先生から課題が与えられて、自分でプログラミングを作らないといけない状態になり必死に頑張りました。あれも違う、これも違うと色々考えた結果、プログラムが正常に動いた時はすごく嬉しかったです。

卒業研究の途中でわからない所が多すぎて飯高先生に質問ばかりしてしまいましたが、嫌な顔せずに教えてくれて、また証明する際に回り道した時も一緒に真剣に考えてくれて、とても感謝しています。ありがとうございました。

1 年間、長いようで意外とあっという間でした。卒業研究を飯高ゼミで出来て良かったです。本当にお世話になりました。